Mixed Designs: Between and Within

Psy 420
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Mixed Between and Within Designs

- Conceptualizing the Design
  - Types of Mixed Designs
- Assumptions
- Analysis
  - Deviation
  - Computation
- Higher order mixed designs
- Breaking down significant effects

Conceptualizing the Design

- This is a very popular design because you are combining the benefits of each design
- Requires that you have one between groups IV and one within subjects IV
- Often called “Split-plot” designs, which comes from agriculture
- In the simplest 2 x 2 design you would have

Conceptualizing the Design

- In the simplest 2 x 2 design you would have subjects randomly assigned to one of two groups, but each group would experience 2 conditions (measurements)

	GRE - before	GRE - after
Kaplan	S_1	S_1
	S_2	S_2
	S_3	S_3
	S_4	S_4
	S_5	S_5
Princeton	S_6	S_6
	S_7	S_7
	S_8	S_8
	S_9	S_9
	S_{10}	S_{10}

Conceptualizing the Design

- Advantages
  - First, it allows generalization of the repeated measures over the randomized groups levels
  - Second, reduced error (although not as reduced as purely WS) due to the use of repeated measures
- Disadvantages
  - The addition of each of their respective complexities

Conceptualizing the Design

	Pretest		Posttest
Treatment Group	S_1	Treatment	S_1
	S_2		S_2
	S_3		S_3
	S_4		S_4
	S_5		S_5
Control Group	S_6	No Treatment	S_6
	S_7		S_7
	S_8		S_8
	S_9		S_9
	S_{10}		S_{10}

- Types of Mixed Designs
  - Other than the mixture of any number of BG IVs and any number of WS IVs…
  - Pretest Posttest Mixed Design to control for testing effects

Assumptions

- Normality of Sampling Distribution of the BG IVs
  - Applies to the case averages (averaged over the WS levels)
- Homogeneity of Variance
  - Applies to every level or combination of levels of the BG IV(s)

Assumptions

- Independence, Additivity, Sphericity
  - Independence applies to the BG error term
  - But each WS error term confounds random variability with the subjects by effects interaction
  - So we need to test for sphericity instead; the test is on the average variance/covariance matrix (over the levels of the BG IVs)

Assumptions

- Outliers
  - Look for them in each cell of the design
- Missing data
  - Causes the same problems that they did in the BG and WS designs separately
    - Data points missing across the WS part can be estimated as discussed previously
    - Missing data in the randomized groups part causes non-orthogonality

Analysis

		Within Groups		
		b_1	b_2	b_3
Randomized Groups	a_1	S_1	S_1	S_1
		S_2	S_2	S_2
		S_3	S_3	S_3
		S_4	S_4	S_4
		S_5	S_5	S_5
	a_2	S_6	S_6	S_6
		S_7	S_7	S_7
		S_8	S_8	S_8
		S_9	S_9	S_9
		S_{10}	S_{10}	S_{10}
	a_3	S_{11}	S_{11}	S_{11}
		S_{12}	S_{12}	S_{12}
		S_{13}	S_{13}	S_{13}
		S_{14}	S_{14}	S_{14}
		S_{15}	S_{15}	S_{15}

Sources of Variance

- $SS_T = SS_{BG} + SS_{WS}$
- What are the sources of variance?
  - A
  - S/A
  - B
  - AB
  - BxS/A
  - T
- Degrees of freedom?

Example – Books by Month

- Example:
  - Imagine if we designed the previous research study concerning reading different novels over time
  - But instead of having everyone read all of the books for three months we randomly assign subjects to three different books and have them read for three months

			B: *Month*			
			b_1: *Month 1*	b_2: *Month 2*	b_3: *Month 3*	Case Means
A: *Type of Novel*	a_1: *Science Fiction*	S_1	1	3	6	**$S_1 = 3.333$**
		S_2	1	4	8	**$S_2 = 4.333$**
		S_3	3	3	6	**$S_3 = 4$**
		S_4	5	5	7	**$S_4 = 5.667$**
		S_5	2	4	5	**$S_5 = 3.667$**
			$a_1b_1 = 2.4$	**$a_1b_2 = 3.8$**	**$a_1b_3 = 6.4$**	**$a_1 = 4.2$**
	a_2: *Mystery*	S_6	3	1	0	**$S_6 = 1.333$**
		S_7	4	4	2	**$S_7 = 3.333$**
		S_8	5	3	2	**$S_8 = 3.333$**
		S_9	4	2	0	**$S_9 = 2$**
		S_{10}	4	5	3	**$S_{10} = 4$**
		a2	**$a_2b_1 = 4$**	**$a_2b_2 = 3$**	**$a_2b_3 = 1.4$**	**$a_2 = 2.8$**
	a_3: *Romance*	S_{11}	4	2	0	**$S_{11} = 2$**
		S_{12}	2	6	1	**$S_{12} = 3$**
		S_{13}	3	3	3	**$S_{13} = 3$**
		S_{14}	6	2	1	**$S_{14} = 3$**
		S_{15}	3	3	2	**$S_{15} = 2.667$**
			$a_3b_1 = 3.6$	**$a_3b_2 = 3.2$**	**$a_3b_3 = 1.4$**	**$a_3 = 2.733$**
			$b_1 = 3.333$	**$b_2 = 3.333$**	**$b_3 = 3.067$**	**GM = 3.244**

Sums of Squares - Deviation

- The total variability can be partitioned into A, B, AB, S/A, and B*S/A

$$SS_{Total} = SS_A + SS_B + SS_{AB} + SS_{S/A} + SS_{B*S/A}$$

$$\sum Y_{ijk} - \bar{Y}_{...}{}^2 = \sum n_j \; \bar{Y}_{.j.} - \bar{Y}_{...}{}^2 + \sum n_k \; \bar{Y}_{..k} - \bar{Y}_{...}{}^2 +$$

$$+\left[\sum n_{jk} \; \bar{Y}_{.jk} - \bar{Y}_{...}{}^2 - \sum n_j \; \bar{Y}_{.j.} - \bar{Y}_{...}{}^2 - \sum n_k \; \bar{Y}_{..k} - \bar{Y}_{...}{}^2\right]$$

$$+ j\sum \; \bar{Y}_{i..} - \bar{Y}_{.j.}{}^2 + \left[\sum \; Y_{ijk} - \bar{Y}_{.jk}{}^2 - j\sum \; \bar{Y}_{i..} - \bar{Y}_{.k.}{}^2\right]$$

$$SS_A = \sum n_j \left(\bar{Y}_{.j.} - \bar{Y}_{...} \right)^2 = 15*[\ (4.2 - 3.244)^2 + (2.8 - 3.244)^2 + (2.733 - 3.244)^2\ ] = 20.583$$

$$SS_B = \sum n_k \left(\bar{Y}_{..k} - \bar{Y}_{...} \right)^2 = 15*[\ (3.333 - 3.244)^2 + (3.333 - 3.244)^2 + (3.067 - 3.244)^2\ ] = .708$$

$$SS_{AB} = \left[\sum n_{jk} \left(\bar{Y}_{.jk} - \bar{Y}_{...} \right)^2 - \sum n_j \left(\bar{Y}_{.j.} - \bar{Y}_{...} \right)^2 - \sum n_k \left(\bar{Y}_{..k} - \bar{Y}_{...} \right)^2 \right] =$$

$$\sum n_{jk} \left(\bar{Y}_{.jk} - \bar{Y}_{...} \right)^2 = 5*[\ (2.4 - 3.244)^2 + (3.8 - 3.244)^2 + (6.4 - 3.244)^2 +$$

$$+ (4 - 3.244)^2 + (3 - 3.244)^2 + (1.4 - 3.244)^2 +$$

$$+ (3.6 - 3.244)^2 + (3.2 - 3.244)^2 + (1.4 - 3.244)^2\ ] = 92.711$$

$$SS_{AB} = 92.711 - 20.583 - .708 = 71.420$$

$$SS_{S/A} = k\sum \left(\bar{Y}_{i..} - \bar{Y}_{.j.} \right)^2 = 3*[\ 3.333-4.2\ ^2 +\ 4.333-4.2\ ^2 +\ 4-4.2\ ^2 +$$

$$+\ 5.667-4.2\ ^2 +\ 3.667-4.2\ ^2 +\ 3.333-2.8\ ^2 +\ 1.333-2.8\ ^2 +\ 3.333-2.8\ ^2 +$$

$$+\ 3.333-2.8\ ^2 +\ 2-2.8\ ^2 +\ 2-2.733\ ^2 +\ 3-2.733\ ^2 +\ 3-2.733\ ^2 +$$

$$+\ 3-2.733\ ^2 +\ 2.667-2.733\ ^2] = 26.400$$

$$SS_{B*S/A} = \left[\sum Y_{ijk} - \bar{Y}_{.jk}^{\ 2} - k \sum \bar{Y}_{i..} - \bar{Y}_{.j.}^{\ 2} \right] =$$

$$\sum Y_{ijk} - \bar{Y}_{.jk}^{\ 2} = 1-2.4^{2} + 1-2.4^{2} + 3-2.4^{2} + 5-2.4^{2} + 2-2.4^{2} +$$

$$+ 3-3.8^{2} + 4-3.8^{2} + 3-3.8^{2} + 5-3.8^{2} + 4-3.8^{2} +$$

$$+ 6-6.4^{2} + 8-6.4^{2} + 6-6.4^{2} + 7-6.4^{2} + 5-6.4^{2} +$$

$$+ 3-4^{2} + 4-4^{2} + 5-4^{2} + 4-4^{2} + 4-4^{2} +$$

$$+ 1-3^{2} + 4-3^{2} + 3-3^{2} + (2-3)^{2} + (6-3)^{2} +$$

$$+ (0-1.4)^{2} + (2-1.4)^{2} + (2-1.4)^{2} + (0-1.4)^{2} + (3-1.4)^{2} +$$

$$+ (4-3.6)^{2} + (2-3.6)^{2} + (3-3.6)^{2} + (6-3.6)^{2} + (3-3.6)^{2} +$$

$$+ (2-3.2)^{2} + (6-3.2)^{2} + (3-3.2)^{2} + (2-3.2)^{2} + (3-3.2)^{2} +$$

$$+ (0-1.4)^{2} + (1-1.4)^{2} + (3-1.4)^{2} + (1-1.4)^{2} + (2-1.4)^{2} = 63.6$$

$$SS_{B*S/A} = 63.6 - 26.4 = 37.2$$

$$SS_{Total} = \sum \left(Y_{ijk} - \bar{Y}_{...} \right)^2 =$$

$$SS_{Total} = (1-3.244)^2 + (1-3.244)^2 + (3-3.244)^2 + (5-3.244)^2 + (2-3.244)^2 +$$

$$+ (3-3.244)^2 + (4-3.244)^2 + (3-3.244)^2 + (5-3.244)^2 + (4-3.244)^2 +$$

$$+ (6-3.244)^2 + (8-3.244)^2 + (6-3.244)^2 + (7-3.244)^2 + (5-3.244)^2 +$$

$$+ (3-3.244)^2 + (4-3.244)^2 + (5-3.244)^2 + (4-3.244)^2 + (4-3.244)^2 +$$

$$+ (1-3.244)^2 + (4-3.244)^2 + (3-3.244)^2 + (2-3.244)^2 + (6-3.244)^2 +$$

$$+ (0-3.244)^2 + (2-3.244)^2 + (2-3.244)^2 + (0-3.244)^2 + (3-3.244)^2 +$$

$$+ (4-3.244)^2 + (2-3.244)^2 + (3-3.244)^2 + (6-3.244)^2 + (3-3.244)^2 +$$

$$+ (2-3.244)^2 + (6-3.244)^2 + (3-3.244)^2 + (2-3.244)^2 + (3-3.244)^2 +$$

$$+ (0-3.244)^2 + (1-3.244)^2 + (3-3.244)^2 + (1-3.244)^2 + (2-3.244)^2 = 156.311$$

			B: *Month*			
			b_1: *Month 1*	b_2: *Month 2*	b_3: *Month 3*	Case Total
A: *Type of Novel*	a_1: *Science Fiction*	S_1	1	3	6	**$S_1 = 10$**
		S_2	1	4	8	**$S_2 = 13$**
		S_3	3	3	6	**$S_3 = 12$**
		S_4	5	5	7	**$S_4 = 17$**
		S_5	2	4	5	**$S_5 = 11$**
			$a_1b_1 = 12$	**$a_1b_2 = 19$**	**$a_1b_3 = 32$**	**$a_1 = 63$**
	a_2: *Mystery*	S_6	3	1	0	**$S_6 = 4$**
		S_7	4	4	2	**$S_7 = 10$**
		S_8	5	3	2	**$S_8 = 10$**
		S_9	4	2	0	**$S_9 = 6$**
		S_{10}	4	5	3	**$S_{10} = 12$**
			$a_2b_1 = 20$	**$a_2b_2 = 15$**	**$a_2b_3 = 7$**	**$a_2 = 42$**
	a_3: *Romance*	S_{11}	4	2	0	**$S_{11} = 6$**
		S_{12}	2	6	1	**$S_{12} = 9$**
		S_{13}	3	3	3	**$S_{13} = 9$**
		S_{14}	6	2	1	**$S_{14} = 9$**
		S_{15}	3	3	2	**$S_{15} = 8$**
			$a_3b_1 = 18$	**$a_3b_2 = 16$**	**$a_3b_3 = 7$**	**$a_3 = 41$**
			$b_1 = 50$	**$b_2 = 50$**	**$b_3 = 46$**	**Total = 146**

Sums of Squares - Computational

- What are the degrees of freedom?
- And convert them into the formulas
  - $A = a - 1$
  - $S/A = a(s - 1) = as - a$
  - $B = b - 1$
  - $AB = (a - 1)(b - 1)$
  - $BxS/A = a(b - 1)(s - 1)$
  - $T = abs - 1$ or $N - 1$

$$SS_A = \frac{\sum A^2}{bs} - \frac{T^2}{abs} = \frac{63^2 + 42^2 + 41^2}{3(5)} - \frac{146^2}{3(3)(5)}$$

$$SS_{S/A} = \frac{\sum (AS)^2}{b} - \frac{\sum A^2}{bs} = \frac{10^2 + 13^2 + 12^2 + \cdots + 8^2}{3} - \frac{63^2 + 42^2 + 41^2}{3(5)}$$

$$SS_B = \frac{\sum B^2}{as} - \frac{T^2}{abs} = \frac{50^2 + 50^2 + 46^2}{3(5)} - \frac{146^2}{3(3)(5)}$$

$$SS_{AB} = \frac{\sum (AB)^2}{s} - \frac{\sum A^2}{bs} - \frac{\sum B^2}{as} + \frac{T^2}{abs} = \frac{12^2 + 19^2 + 32^2 + 20^2 + 15^2 + 7^2 + 18^2 + 16^2 + 7^2}{5} - \frac{63^2 + 42^2 + 41^2}{3(5)} - \frac{50^2 + 50^2 + 46^2}{3(5)} + \frac{146^2}{3(3)(5)}$$

$$SS_{B\times S/A} = \sum Y^2 - \frac{\sum (AB)^2}{s} - \frac{\sum (AS)^2}{b} + \frac{\sum A^2}{bs} = 1^2 + 1^2 + 3^2 + 5^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots + 1^2 + 2^2 - \frac{12^2 + 19^2 + 32^2 + 20^2 + 15^2 + 7^2 + 18^2 + 16^2 + 7^2}{5} - \frac{10^2 + 13^2 + \cdots + 8^2}{3} + \frac{63^2 + 42^2 + 41^2}{3(5)}$$

$$SS_T = \sum Y^2 - \frac{T^2}{abs} = 1^2 + 1^2 + 3^2 + 5^2 + 2^2 + 3^2 + \cdots + 1^2 + 2^2 - \frac{146^2}{3(3)(5)}$$

$$
\begin{aligned}
SS_A &= 494.27 - 473.69 &&= 20.58 \\
SS_{S/A} &= 520.67 - 494.27 &&= 26.40 \\
SS_B &= 474.40 - 473.69 &&= 0.71 \\
SS_{AB} &= 566.40 - 494.27 - 474.40 + 473.69 &&= 71.42 \\
SS_{B\times S/A} &= 630 - 566.40 - 520.67 + 494.27 &&= 37.20 \\
SS_T &= 630 - 473.69 &&= 156.31
\end{aligned}
$$

$$df_A = a - 1 = 3 - 1 = 2$$

$$df_{S/A} = a(s - 1) = 3(5 - 1) = 12$$

$$df_B = b - 1 = 3 - 1 = 2$$

$$df_{AB} = (a - 1)(b - 1) = (3 - 1)(3 - 1) = 4$$

$$df_{BxS/A} = a(b - 1)(s - 1) = 3(3 - 1)(5 - 1) = 24$$

$$df_T = abs - 1 = N - 1 = 3(3)(5) - 1 = 44$$

Results – ANOVA summary table

Source	SS	df	MS	*F*
Randomized Groups				
A	20.58	2	10.29	$\frac{10.29}{2.20} = 4.68$
S/A	26.40	12	2.20	
Repeated Measures				
B	0.71	2	0.36	$\frac{0.36}{1.55} = 0.23$
$A \times B$	71.42	4	17.86	$\frac{17.86}{1.55} = 11.52$
$B \times S/A$	37.20	24	1.55	
T	156.31	44		

Higher order mixed designs

Design	Randomized-Groups IVs	Repeated-Measures IVs	Sources of Variability		
			Randomized Groups	*Repeated Measures*	*Error Terms*
Two-way mixed	A	B	A		S/A
				$B, A \times B$	$B \times S/A$
Three-way mixed	A, B	C	$A, B, A \times B$		S/AB
				$C, A \times C, B \times C, A \times B \times C$	$C \times S/AB$
	A	B, C	A		S/A
				$B, A \times B$	$B \times S/A$
				$C, A \times C$	$C \times S/A$
				$B \times C, A \times B \times C$	$B \times C \times S/A$
Four-way mixed	A, B	C, D	$A, B, A \times B$		S/AB
				$C, A \times C, B \times C, A \times B \times C$	$C \times S/AB$
				$D, A \times D, B \times D, A \times B \times D$	$D \times S/AB$
				$C \times D, A \times C \times D,$ $B \times C \times D, A \times B \times C \times D$	$C \times D \times S/AB$

Breaking down significant effects

- Between Groups IV(s)
  - If you have a significant BG main effect(s) they need to be broken down to find which levels are different
  - The comparisons are done the same way as completely BG comparisons
  - The BG comparison error term is the same for all BG comparisons

$$F = \frac{n_{\bar{Y}}(\sum w_j \bar{Y}_j)^2 / \sum w_j^2}{MS_{S/AB\cdots}} = \frac{SS_{(reg.X_j)}}{MS_{S/AB}}$$

Breaking down significant effects

- Within Groups Variables
  - If a WG main effect is significant it also needs to be followed by comparisons
  - WG comparisons differ from BG variables in that a separate error term needs to be generated for each comparison
  - Instead of the Fcomp formula you would actually rearrange the data into a new data set

Breaking down significant effects

- Example

		***B*: Month**			
		b_1: *Month 1*	b_2: *Month 2*	b_3: *Month 3*	**Case Totals**
a_1: Science Fiction	s_1	1	3	6	$A_1S_1 = 10$
	s_2	1	4	8	$A_1S_2 = 13$
	s_3	3	3	6	$A_1S_3 = 12$
	s_4	5	5	7	$A_1S_4 = 17$
	s_5	2	4	5	$A_1S_5 = 11$
		$A_1B_1 = 12$	$A_1B_2 = 19$	$A_1B_3 = 32$	$A_1 = 63$
a_2: Mystery	s_6	3	1	0	$A_2S_6 = 4$
	s_7	4	4	2	$A_2S_7 = 10$
	s_8	5	3	2	$A_2S_8 = 10$
	s_9	4	2	0	$A_2S_9 = 6$
	s_{10}	4	5	3	$A_2S_{10} = 12$
		$A_2B_1 = 20$	$A_2B_2 = 15$	$A_2B_3 = 7$	$A_2 = 42$
a_3: Romance	s_{11}	4	2	0	$A_3S_{11} = 6$
	s_{12}	2	6	1	$A_3S_{12} = 9$
	s_{13}	3	3	3	$A_3S_{13} = 9$
	s_{14}	6	2	1	$A_3S_{14} = 9$
	s_{15}	3	3	2	$A_3S_{15} = 8$
		$A_3B_1 = 18$	$A_3B_2 = 16$	$A_3B_3 = 7$	$A_3 = 41$
		$B_1 = 50$	$B_2 = 50$	$B_3 = 46$	$T = 146$

Breaking down significant effects

		B			
		b_1	b_3		
a_1	S_1	1	6	S1 = 7	
	S_2	1	8	S2 = 9	
	S_3	3	6	S3 = 9	
	S_4	5	7	S4 = 12	
	S_5	2	5	S5 = 7	
		a1b1=12	a1b3=32	a1=44	
a_2	S_6	3	0	S6 = 3	
	S_7	4	2	S7 = 6	
	S_8	5	2	S8 = 7	
	S_9	4	0	S9 = 4	
	S_{10}	4	3	S10 = 7	
		a2b1=20	a2b3=7	a2=27	
a_3	S_{11}	4	0	S11 = 4	
	S_{12}	2	1	S12 = 3	
	S_{13}	3	3	S13 = 6	
	S_{14}	6	1	S14 = 7	
	S_{15}	3	2	S15 = 5	
		a3b1=18	a3b3=7	a3=25	T = 96

Breaking down significant effects

- Interactions
  - Purely BG interactions can be treated with simple effects, simple contrasts and interaction contrasts using the Fcomp formula, the same error term each time
  - Purely WG and mixed BG/WG interactions require a new error term for each simple effect, simple contrast and interaction contrast (leave it to SPSS)